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NATURAL CONVECTION FROM A CYLINDER IN A NARROW GAP AND IN A POROUS MEDIUM

A. V. Gorin, A. Yu. Egorov,
V. E. Nakoryakov, and V. M. Chupin

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The problem of natural convection from a horizontal cylinder in a narrow gap and in a porous medium is solved both theoretically and experimentally. An integral method for calculating heat transfer from the cylinder for constant flux on its surface was suggested. Numerical analysis clarified the role of regime and geometrical factors. It is shown that natural convection from a cylinder in a porous medium can be modeled by a Hele Shaw cell.

The steady interest, over several decades, in problems of heat and mass transfer in granular and porous media and the development of a quantitative theory and methods of experimental investigation and simulation of transfer processes in these media have been motivated by the wide occurrence of these media both in nature and in modern technologies. For example, the solution of heat-transfer problems for natural convection from a horizontal cylinder embedded in a porous medium is directly related to the development of methods of activating heat- and mass-transfer processes, the design of compact heat exchangers and heat insulators, oil production, burial of radioactive waste, etc.

Fand et al. [1] obtained a self-similar solution of the problem of convective heat transfer for an isothermal horizontal cylinder embedded in a porous medium in a boundary-layer approximation. Schrock et al. [2] and Ferkandez and Schrock [3] performed perhaps the first experimental studies of this problem, which indicated the important role of geometrical factors such as the ratio of the depth of an embedded cylinder H to its diameter $D = 2R$ and length L. Fand et al. [1] performed experiments on natural convection from a horizontal cylinder embedded in a granular medium over a broad range of Rayleigh numbers and undertook an attempt to take into account the viscous dissipation of filtration flow in experimental-data processing. It is noted that a change in the drag law leads to changes in the heat-transfer law.

Since experiments in a porous medium are labor consuming and experimental techniques are limited, continuous searches for methods of modeling transfer processes in a porous medium have been conducted. On the basis that the mathematical model of laminar flow in a narrow gap between parallel plates (a Hele Shaw cell) agrees with filtration equations with a linear drag law (the Darcy law), Ber et al. [4] used a Hele Shaw cell to model the hydrodynamic characteristics of liquid filtration in a porous medium.

The theoretical and experiments results of [5–7] for natural convection on the side wall of a Hele Shaw cell formed by adiabatic plates showed that natural convection heat transfer on a vertical plate embedded in a porous medium can be modeled by a Hele Shaw cell. Use of a boundary-layer approximation enabled self-similar solutions to be obtained for the corresponding type of boundary conditions.

Hele Shaw devices are conveniently and effectively used in experimental studies, including visualization, in contrast to a porous medium, for which studies by contact methods inevitably lead to violation of its structure. Noncontact methods such as Doppler laser anemometry are adequate only for optical liquids with strict control over the temperature regime, and this strongly limits the applicability of these methods in heat- and mass-transfer studies.

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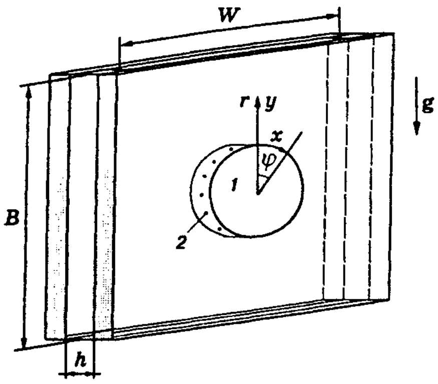

Fig. 1

The solution of heat- and mass-transfer problems in narrow gaps is of obviously importance for cooling of modern radioelectronic equipment with highly integrated components, intense heat release, and the trend to miniaturization.

In the work presented here, natural convection from a horizontal cylinder in a porous medium and in a narrow gap is studied both theoretically and experimentally. An integral method for calculating heat transfer from the cylinder with constant heat flux on its surface is suggested. The results are compared with literature data on heat exchange for a cylinder imbedded in a porous medium. Numerical analysis enabled us to clarify the role of the regime and geometrical parameters.

1. Formulation of the Problem. Steady flow and natural convection heat transfer of a heated horizontal cylinder of radius R placed in a Hele Shaw cell formed by adiabatic vertical flat walls separated by a distance h (Fig.1) is described by the system

$$\rho(\mathbf{V}\,\nabla)\,\mathbf{V} = \mu\Delta\mathbf{V} - \nabla P + \rho_0\beta(T - T_0)\,\mathbf{g}, \quad \rho C_p(\mathbf{V}\nabla)T = \lambda\Delta T, \quad \operatorname{div}\mathbf{V} = 0, \tag{1.1}$$

where λ, μ, and C_p are the constant thermal conductivity, dynamic viscosity, and specific heat of the liquid, ρ_0 is the liquid density at the temperature at infinity T_0, T and P are the temperature and pressure, respectively, $\mathbf{V}$ is the velocity vector, $\mathbf{g}$ is the acceleration of gravity, and $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplacian operator.

Assuming the absence of transverse motion, one can use a parabolic velocity distribution and constant temperature across the cell. This is justified by the smallness of the gap $h \ll R$ and low velocities of motion:

$$V_r(r,\varphi,z) = \frac{3}{2}\,V_r(r,\varphi)\Big(1 - 4\,\frac{z^2}{h^2}\Big), \quad V_\varphi(r,\varphi,z) = \frac{3}{2}\,V_\varphi(r,\varphi)\Big(1 - 4\,\frac{z^2}{h^2}\Big); \tag{1.2}$$

$$T(r,\varphi,z) = T(r,\varphi). \tag{1.3}$$

Substituting (1.2) and (1.3) into system (1.1) and integrating it from $-h/2$ to $h/2$ over z similarly to [5–7], we obtain the two-dimensional problem.

In this paper, the following two types of boundary conditions are analyzed:

- isothermal surface of the cylinder ($T_w = \text{const}$);
- constant flux on the cylinder surface ($q_w = \text{const}$).

Even in the two-dimensional formulation, the solution of the problem requires the use of numerical methods. First of all, let us perform a qualitative analysis and obtain simple approximate solutions using obvious physical simplifications.

Dimensional Analysis. Let us consider the problem of natural convection from the heated cylinder in a porous medium with permeability K (in the case of a Hele Shaw cell, $K = h^2/12$) for the Darcy filtration regime in the approximation of a thick boundary layer of thickness $\delta \ll R$:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2},\qquad u=\frac{g\beta K\Delta T}{\nu}\sin\frac{x}{R},\qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1.4}$$

Here the coordinates x and y (Fig. 1) are related to the cylindrical coordinates r and φ by the equations $x = R\varphi$ and $y = r - R$. In system (1.4) and in what follows, $\Delta T = T - T_0$.

For a porous medium the thermal conductivity a and kinematic viscosity ν have the meaning of effective transfer coefficients, whereas for a Hele Shaw cell these are the physical properties of the medium. From the continuity equation it follows that $u/R \sim v/\delta$ or $v \sim u\delta/R$.

The first equation of system (1.4) leads to the estimate

$$u\frac{\Delta T}{R},\ v\frac{\Delta T}{\delta}\sim a\frac{\Delta T}{\delta^2}\quad\Rightarrow\quad v\sim\frac{a}{\delta},\quad u\sim a\frac{R}{\delta^2}. \tag{1.5}$$

Here and below, u, v, ΔT, and δ denote the scales of the corresponding quantities. In the case of an isothermal cylinder, using the second equation of system (1.4), for the boundary-layer thickness we have $\delta/R \sim \mathrm{Ra}^{-1/2}$, where the Rayleigh number is $\mathrm{Ra} = g\beta K\Delta TR/(\nu a)$.

From the definition of the Nusselt number we obtain

$$\mathrm{Nu}=\frac{\alpha D}{\lambda}\sim\frac{q_w R}{\lambda\Delta T}\sim\frac{\lambda(\Delta T/\delta)R}{\lambda\Delta T}\sim\left(\frac{\delta}{R}\right)^{-1},$$

i.e.,

$$\mathrm{Nu}_T\sim\mathrm{Ra}_T^{1/2}. \tag{1.6}$$

Here and below, the subscript T refers to the isothermal cylinder and the subscript q characterizes the constant flux problem.

In [1], the heat-transfer law (1.6) is obtained by a self-similar solution of the natural convection heat-transfer problem in a boundary layer approximation for an isothermal cylinder embedded in a porous medium and has the form

$$\mathrm{Nu}_T = 0.565\,\mathrm{Ra}_T^{1/2}. \tag{1.7}$$

In [1], D was used as the characteristic linear dimension.

In the case of constant flux q_w on the cylinder surface, the temperature scale (from the definition of heat flux) is estimated as $\Delta T \sim q_w\delta/\lambda$.

Then, with allowance for (1.5) we have

$$\frac{g\beta K}{\nu}\frac{q_w\delta}{\lambda}\sim a\frac{R}{\delta^2}\quad\Rightarrow\quad\frac{\delta}{R}\sim\mathrm{Ra}_q^{-1/3},$$

where $\mathrm{Ra}_q = q\beta KR^2q_w/(a\nu\lambda)$. Hence, for the heat-transfer law we obtain

$$\mathrm{Nu}_q\sim\mathrm{Ra}_q^{1/3}. \tag{1.8}$$

One can also obtain the heat-transfer laws for the limiting case of dominance of flow inertial effects, i.e., under conditions of violation of the Darcy law. Let us write the drag law in the form

$$\frac{\mu}{K}u+\frac{\rho C}{\sqrt{K}}u^2=\rho g\beta\Delta T.$$

Usually, $C = 0.5$. From the drag law it follows that

$$u=\frac{K}{\nu}g\beta\Delta T\,\frac{\sqrt{1+4\mathrm{Gr}_I}-1}{2\mathrm{Gr}_I}=u_L\,\frac{\sqrt{1+4\mathrm{Gr}_I}-1}{2\mathrm{Gr}_I},$$

where Gr_I is the Grashof number ($\mathrm{Gr}_I = g\beta\Delta TCK^{3/2}/\nu^2$) and $u_L = (K/\nu)g\beta\Delta T$.

In the limiting case where inertial effects are dominant, we have $u \approx u_L/\mathrm{Gr}_I$. In this case, dimensional analysis gives

$$u \sim \left(\frac{g\beta\delta\sqrt{K}}{\lambda}\right)^{1/2} \sim a\,\frac{R}{\delta^2},$$

and, hence, $\delta/R \sim (\mathrm{Da}/(\mathrm{Ra}_q\mathrm{Pr}))^{-1/5}$, where $\mathrm{Ra}_q = g\beta q_w K R^2/(\nu a\lambda)$, $\mathrm{Pr} = \nu/a$, and $\mathrm{Da} = \sqrt{K}/R$. Then

$$\mathrm{Nu}_q \sim (\mathrm{Ra}_q\mathrm{Pr}/\mathrm{Da})^{1/5}.$$

For an isothermal cylinder, one can similarly obtain the following heat-transfer law:

$$\mathrm{Nu}_T \sim (\mathrm{Ra}_T\mathrm{Pr}/\mathrm{Da})^{1/4}.$$

Thus, dimensional analysis enabled us to obtain the form of the basic heat-transfer laws for a cylinder embedded in a porous medium under natural-convection conditions.

Integral Method. The problem with constant heat flux q_w on the cylinder surface, in contrast to the isothermal case, does not have a self-similar solution. Nevertheless, using the integral, from (1.4) one can obtain an approximate solution that would specify expression (1.8).

Let us integrate the energy equation over the thickness of the thermal boundary layer:

$$\frac{\partial}{\partial x}\int_0^\delta uT\,dy = \frac{q_w}{\rho C_p}. \tag{1.9}$$

We write the temperature distribution in the form $T(x,y) = X(x)\Theta(\eta)$, where $\eta = y/\delta$. Then, Eq. (1.9) can be rewritten in the new variables as

$$\frac{q_w x}{\rho C_p} = c_1\frac{g\beta K\delta}{\nu}X^2\sin\frac{x}{R},$$

where

$$c_1 = \int_0^1 \Theta^2\,d\eta.$$

Defining the boundary-layer thickness δ as

$$q_w = \lambda\,\frac{\partial T}{\partial y}\bigg|_{y=0} = \lambda\,\frac{X}{\delta_T}\,\frac{d\Theta}{d\eta}\bigg|_{\eta=0} \quad\Rightarrow\quad \delta_T = -\frac{\lambda\Theta'(0)}{q_w}X,$$

we obtain

$$\frac{q_w x}{\rho C_p} = c_1\,\frac{g\beta K}{\nu}\,\frac{\lambda\Theta'(0)}{q_w}\,X^3\sin\frac{x}{R}.$$

Whence

$$X^3 = \Delta T^3\,\frac{\lambda^2\nu}{8R^2 q_w\rho C_p g\beta K c_1\Theta'(0)}\,\frac{\varphi}{\sin\varphi},$$

where $\Delta T = q_w D/\lambda$. Taking into account that $\mathrm{Nu} = \Delta T/X$, we obtain

$$\mathrm{Nu}(\varphi) = -[2c_1\Theta'(0)]^{1/3}\mathrm{Ra}_q^{1/3}(\sin\varphi/\varphi)^{1/3}.$$

Here and below, the cylinder diameter is used in both experiments and calculations of the Rayleigh and Nusselt numbers.

For the Nusselt number averaged over the cylinder surface, we have

$$\overline{\mathrm{Nu}} = 0.7878[-2c_1\Theta'(0)]^{1/3}\mathrm{Ra}_q^{1/3}.$$

If we set $\Theta = (1-\eta)^2$, we obtain

$$\overline{\mathrm{Nu}} = 0.731\,\mathrm{Ra}_q^{1/3}. \tag{1.10}$$

The dimensional analysis and the solution obtained by the integral method described above make it possible to analyze the heat transfer from the cylinder placed in a Hele Shaw cell or in a porous medium under natural-convection conditions and with constant heat flux.

The detailed structure of the flow field and allowance for the influence of various parameters is provided for by a numerical solution of the problem. A two-dimensional system of equations in the variables Ψ–Ω was used. It was approximated by an integrointerpolation method similar to [8] and was solved by the method of stabilizing correction. The mesh step at the cylinder surface and the dimension of the computation region were determined by test calculations.

2. Experimental Setup and Measurement Technique. The experimental setup (Fig. 1) consisted of cylinder 1 of diameter $D = 40$ mm placed between two 60-mm-thick parallel vertical plates made of foam plastic, whose inner surface was glued around with glass textolite. The working cavity had the following dimensions: height $B = 760$ mm, width $W = 260$ mm, and thickness $h = 2$ mm. Heat was removed by a cooler located in the upper part of the working cavity.

The cylinder with constant heat flux was made of foam plastic. The heater (a 0.1-mm-thick Nichrome foil) was glued along the cylinder perimeter. The heater temperature was measured by seven calibrated Nichrome–Constantan thermocouples 2 of 0.1 mm diameter placed at 30° from each other beginning from the lower point of the cylinder. The thermocouples were fixed on the inside of the foil with epoxy resin with copper oxide filler. The heat flux was determined from the electric power supplied to the heater and it varied from 1.39 to 10.1 W/m^2.

The isothermal cylinder was made from an aluminum pipe with outside diameter 40 mm, wall thickness 3.8 mm, and length 5 mm. A Nichrome foil heater was glued inside the cylinder with epoxy resin. A thin layer of the adhesive served as an electric insulator between the aluminum tube and the heater. Six Nichrome–Constantan thermocouples of 0.1 mm diameter were caulked at 36° from each other closer to the tube outer surface to monitor the temperature distribution on the end walls of the cylinder. The first thermocouple was fixed at the lower point of the cylinder. The cylinder ends were thermally insulated with two ebonite sleeves. This design provided for a uniform temperature distribution on the cylinder surface. Experiments were carried out at temperatures from 23 to 55°. The error of temperature distribution over the working surface of the cylinder at $T = 55$°C did not exceed 4%.

The thermocouples were calibrated prior to the experiments. Electric power was supplied to the heater from an adjustable source of dc current. The water temperature in the working cavity T_0 was monitored by a thermocouple placed 20 mm below the cylinder.

Temperature fields were observed by means of an infrared imager. For this, one of the foam plastic plates was replaced by a plate made of 0.8-mm-thick stainless steel. The emissivity was increased by painting the outer surface of the plate with black graphite paint (the technique is described in [7]).

The streamlines were visualized by introducing a luminophore dye into the working cavity from a collector above the cylinder through 15 holes of 0.2 mm diameter. In this case, the walls of the working cavity were made of Plexiglas. Reflected-light photography was performed.

The mean integral temperature $\bar{T}$ over the entire surface of the cylinder was calculated from the measured temperatures under the surface condition $q_w =$ const. The heat flow Q was determined by the electric power delivered to the heater.

The average Nusselt number was determined from the formulas

$$\mathrm{Nu}_T = \frac{\alpha D}{\lambda} = \frac{QD}{F\lambda(T_w - T_0)}, \qquad \overline{\mathrm{Nu}}_q = \frac{\bar{\alpha} D}{\lambda} = \frac{QD}{F\lambda(\bar{T} - T_0)},$$

where F is the area of the lateral surface of the cylinder.

The values of all physical quantities incorporated in the dimensionless criteria were calculated from the mass mean temperature $T_f = (\bar{T} - T_0)/2$.

3. Analysis of the Results. Numerical calculations were performed for both an isothermal cylinder and for the case of constant flux on the cylinder surface at a Prandtl number $\mathrm{Pr} = 7$. The Rayleigh number was varied in the ranges $1 \leqslant \mathrm{Ra}_T \leqslant 10^4$ and $1 \leqslant \mathrm{Ra}_q \leqslant 10^5$. The Darcy number Da was varied from 0.001 to 0.1.

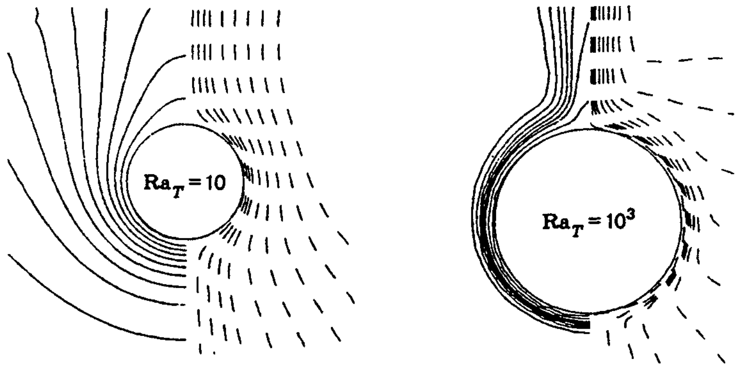

Fig. 2

For a Hele Shaw cell, the Rayleigh and Darcy criteria are defined by

$$\mathrm{Ra}_T = \frac{g\beta\Delta T h^2 D}{12\nu a}, \qquad \mathrm{Ra}_q = \frac{g\beta q_w h^2 D^2}{12\nu a\lambda}, \qquad \mathrm{Da} = \frac{h}{\sqrt{3}D}.$$

Let us discuss some calculation results for natural convection near an isothermal cylinder. In the literature, both theoretical and experimental studies of natural convection near a horizontal cylinder embedded in a porous medium were performed using precisely this type of boundary condition. In addition, we note that for the cases of an isothermal cylinder and constant flux on the cylinder surface, qualitatively similar temperature fields and flow patterns are observed.

At $\mathrm{Ra}_T \sim 1$, the flow is mainly uprising from the cylinder surface and the configuration of the isotherms practically corresponds to a linear heat source. With increase in the Rayleigh number, dynamic and thermal layers with noticeably varying temperature and velocity gradients are formed on the cylinder surface. Their thicknesses are initially comparable with the cylinder diameter, and, hence, the boundary-layer approximation will give a very great error. At $\mathrm{Ra}_T > 10$, the boundary-layer thickness becomes markedly smaller than the cylinder diameter.

Figure 2 shows streamlines (dashed curves) and isotherms (solid curves) for $\mathrm{Ra}_T = 10$ and $\mathrm{Ra}_T = 10^3$ and for $\mathrm{Da} = 0.01$.

The distribution of the angular velocities U_φ is typical of the longitudinal velocities in the boundary layer up to the rear point. The radial velocity U_r is very low and uniform in the inner part of the boundary layer and is directed to the cylinder. At angles $120° < \varphi < 165°$, it changes sign inside the boundary layer. At $\varphi \sim 175°$, the angular and radial velocities are comparable in the magnitude, and the region $175° < \varphi < 180°$ can be considered a region of formation of a thermal jet.

It is worth noting that in our calculations, as in the calculations of [9] for a porous medium, there was no vortex region in the rear part of the cylinder.

The variation in the Darcy number in the calculations did not change the flow pattern and the temperature field qualitatively. Analysis, however, shows that with increase in Da, the boundary layer-thickness grows, and this should be reflected by a decrease in the heat transfer.

Figure 3 shows results of visualization for streamlines (left) and the temperature field (right) for an isothermal cylinder at $\mathrm{Ra}_T = 10^4$. It should be noted that there is fair qualitative agreement between the visualization results and the numerical-calculation values. Some differences in the behavior of isolines in the upper half-plane are due to the finite dimensions of the working cavity. Because of the presence of the upper boundary, the liquid, rising from the heated cylinder and cooling on the cooler, spreads to the side walls of the working cavity and goes down. Thus, a region of double vortex circulation flow is formed above the cylinder. The velocity of the return flow rapidly decreases with decrease in the Rayleigh number. Below the cylinder, unidirectional flow with insignificant liquid ejection from the cell walls to the cylinder surface is observed.

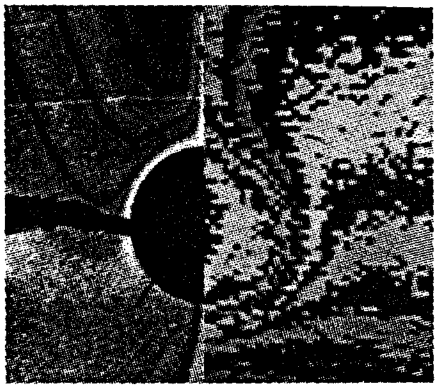

Fig. 3

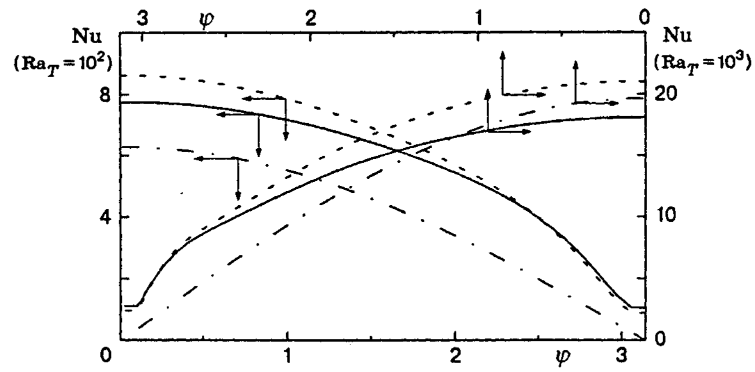

Fig. 4

The condensation of isotherms at the cylinder surface indicates the presence of sharp temperature gradients and a boundary layer in this region. A thermal jet is formed over the cylinder surface. The formation of the thermal jet was observed using the Töpler technique. The experiments showed that for an isothermal cylinder at $Ra_T > 5 \cdot 10^5$, the thermal jet becomes unstable and shifts from the center to the right or left side wall. However, this did not affect the average Nusselt number, and this counts in favor of the experimental technique and geometric dimensions selected. In the case of a cylinder with constant heat flux on the surface, the thermal jet was stable at any values of the Rayleigh number.

A comparison of the flow patterns and temperature fields in natural convection near a horizontal isothermal cylinder in a Hele Shaw cell and near a cylinder embedded at a finite depth [3] in a semiinfinite porous medium, and results of numerical calculations of the same problem for a finite porous medium [9] indicates their good qualitative agreement.

Figure 4 shows the variation of the local Nusselt number over the cylinder circle for various Raleigh and Darcy numbers (the solid curves refer to Da = 0.001 and the dashed curves refer to Da = 0.1) and a self-similar solution [1] (the dot-and-dashed curve) obtained for the boundary-layer approximation. It can be seen that with increase in the Rayleigh number, the boundary-layer approximation becomes more and more adequate for the calculation. The influence of the Darcy number on the calculation results is of methodical importance especially for experimentation: at rather small Rayleigh numbers (~10), a change of Da by two orders of magnitude (from 0.05 to 0.1) does not affect the results, but with increase in the Ra, the differences in the Nusselt numbers become appreciable. For example, at $Ra_T = 10^2$, the heat-transfer coefficients at the front point $\delta = 0$ at Da = 0.05 and Da = 0.1 differ by approximately 10%, but for $Ra_T = 10^3$ this difference already exceeds 20%. This is related to the fact that at relatively small Rayleigh numbers, the boundary-layer

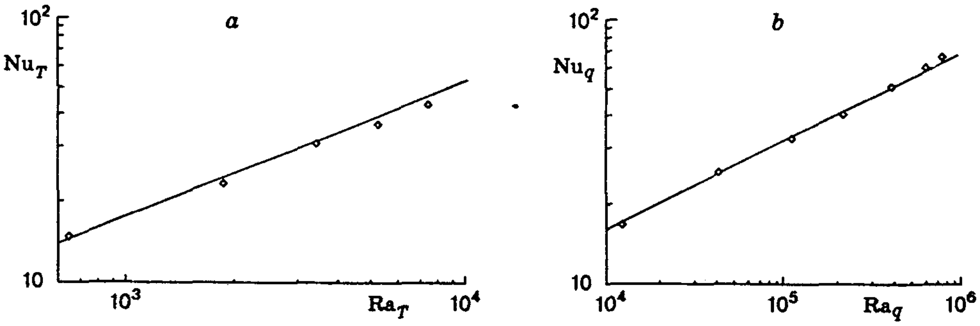

Fig. 5

thickness δ far exceeds the Hele Shaw cell width h and assumptions (1.2) and (1.3) are justified. With increase in Ra, the thickness δ in the vicinity of $\varphi = 0$ becomes comparable with h, and in the averaging of the transfer equations over the cell width, the angular effects should also be taken into account.

Another feature of the effect of the Da revealed in our calculations is that at $\mathrm{Da} = 0.1$ and $\mathrm{Da} = 0.05$, there is a marked difference in results, but a further decrease in Da practically does not affect them.

Figure 5 compares the average Nusselt number (lines) calculated by formula (1.7) for an isothermal cylinder Nu_T (a) and by formula (1.10) for a cylinder with constant flux (b) with experimental results (points) in a Hele Shaw cell. There is good agreement between theory and experiments over the entire study range at $T_w = \mathrm{const}$ and up to $\mathrm{Ra}_q \sim 3 \cdot 10^5$ at $q_w = \mathrm{const}$. At $T_w = \mathrm{const}$, the experimental results were approximated by the equation $\mathrm{Nu}_T = 0.54\,\mathrm{Ra}_T^{1/2}$, which agrees with accuracy up to 4% with the self-similar solution of [1].

For $q_w = \mathrm{const}$, the following approximation holds:

$$\mathrm{Nu}_q = 0.78\,\mathrm{Ra}_q^{1/3}.$$

It corresponds with accuracy up to 6% to the integral solution (1.10) up to $\mathrm{Ra}_q \sim 3 \cdot 10^5$. The deviation of the data for $\mathrm{Ra}_q > 3 \cdot 10^5$ from the law (1.10) is apparently related to the perturbation effect of return flows in the Hele Shaw cell on the boundary layer, which become sufficiently intense at high heat fluxes.

We turn to experimental data on heat transfer from cylinders placed in a porous medium. These data are available only for isothermal cylinders [1].

For natural convection heat transfer in the Darcy regime (the boundary of applicability is given by $\mathrm{Ra} \sim 45$–50), Fand et al. [1] recommend the following approximation:

$$\mathrm{Nu}_T = 0.679\,\mathrm{Ra}_T^{0.646}\mathrm{Pr}^{-0.126}.$$

From this formula it can be seen that the power of Ra far exceeds the theoretical value [see (1.6) and (1.7)]. The diameter of the experimental cylinder was $D = 1.45 \cdot 10^{-2}$ m. Glass beads of diameters $d = 2 \cdot 10^{-3}$, $3 \cdot 10^{-3}$, and $4 \cdot 10^{-3}$ m were used as a porous medium.

At low Rayleigh numbers, the thickness of the thermal boundary layer of the order of the cylinder radius decreases rapidly ($\delta/R \sim \mathrm{Ra}^{-1/2}$) with growth in Ra. This indicates that the dimensions of the beads are comparable to the boundary-layer thickness, and at $\mathrm{Ra} > 10$ they even exceed this value. Thus, the experimental conditions in [1] do not fit the theoretical model of [1] and the one developed in the present paper, and the experimental results cannot be a criterion for the validity of these theories. We also note that allowance for only the increased porosity of the layer of beads near the cylinder surface [1] does not eliminate the main methodical disadvantage of the analysis of the experimental data.

Another set of experimental data is reported by Ferkandez and Schrock [3], who studied the heat transfer of horizontal cylinders of diameters $D = 7.62 \cdot 10^{-2}$ and $6.35 \cdot 10^{-2}$ m embedded in a tank with a water-saturated porous medium (sand). A layer of pure water of constant temperature overlay the sand. The sand particles had mean diameters $d \sim 10^{-3}$ and $0.15 \cdot 10^{-3}$ m.

In [3], based on experiments, the following relation is recommended for the natural-convection heat-transfer coefficient:

$$\mathrm{Nu}_F = f(\eta)\,\mathrm{Ra}_F^{0.514}.$$

The following criteria are used:

$$\mathrm{Nu}_F = \bar{q}D/(T_w - T_s)\lambda_w, \qquad \mathrm{Ra}_F = g\beta\Delta T K D(\rho C_p)_L\sqrt{H^2/R^2 - 1}/(2\nu\lambda_w).$$

Here $\bar{q}$ is the heat flux averaged over the cylinder surface, T_s is the temperature on the surface of the porous medium (the temperature of the liquid layer over the porous medium), and the subscript L denotes the physical properties of the liquid.

The function $f(\eta)$ characterizes the geometry of the problem:

$$f(\eta) = 0.015 + 1.23\,\exp(-0.543\eta), \qquad \eta = \ln\left(\frac{H}{R} + \sqrt{\frac{H^2}{R^2} - 1}\right).$$

Here H is the distance from the granular layer to the center of the cylinder.

According to the data of [3], the convective regime begins with values $\mathrm{Ra}_F > 25$ (from analysis of the figures in [3] it follows that the experiments were conducted up to values of $\mathrm{Ra}_F = 210$).

The thermal conductivity λ and the permeability K of the porous medium were determined in special experiments, and they seem to be adequate provided that $d/R \ll 1$. The experiments of [3] support the validity of (1.7). The function $f(\eta)$ and the choice of T_s on the surface of the porous layer as the reference value in determining ΔT characterize the difference of the experimental conditions from the classical formulation of the heat-transfer problem for an infinite volume of a porous medium.

Thus, the theoretical and experimental studies of flow and heat transfer under natural convection of a horizontal cylinder placed in a Hele Shaw cell are in good agreement. Furthermore, the calculations show that a decrease in the Da number beginning with 0.05 practically does not affect the results.

The known theoretical models and experimental data on natural-convection heat transfer from isothermal horizontal cylinders in porous media generally confirm the similarity between heat-transfer processes in a porous medium in the Darcy filtration regime and in a narrow gap. However, insufficient experimental data does not yet permit one to establish exact limits of applicability for such a similarity. Additional experimental studies of porous media, accurate processing of experimental results for quantities such as the permeability in the wall region and the effective transfer coefficients, especially in comparison with the asymptotic regularities obtained in a boundary-layer approximation, are required.

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